C2 DIFFERENTIATION Worksheet A

1 In each case, find any values of x for which $\frac{dy}{dx} = 0$.

a $y = x^2 + 6x$ **b** $y = 4x^2 + 2x + 1$ **c** $y = x^3 - 12x$ **d** $y = 4 + 9x^2 - x^3$

e $y = x^3 - 5x^2 + 3x$ **f** $y = x + \frac{9}{x}$ **g** $y = (x^2 + 3)(x - 3)$ **h** $y = x^{\frac{1}{2}} - 2x$

2 Find the set of values of x for which f(x) is increasing when

a $f(x) \equiv 2x^2 + 2x + 1$ **b** $f(x) \equiv 3x^2 - 2x^3$ **c** $f(x) \equiv 3x^3 - x - 7$

d $f(x) \equiv x^3 + 6x^2 - 15x + 8$ **e** $f(x) \equiv x(x - 6)^2$ **f** $f(x) \equiv 2x + \frac{8}{x}$

3 Find the set of values of x for which f(x) is decreasing when

a $f(x) \equiv x^3 + 2x^2 + 1$ **b** $f(x) \equiv 5 + 27x - x^3$ **c** $f(x) \equiv (x^2 - 2)(2x - 1)$

4
$$f(x) \equiv x^3 + kx^2 + 3.$$

Given that $(x + 1)$ is a factor of f(x),

a find the value of the constant k,

b find the set of values of x for which f(x) is increasing.

5 Find the coordinates of any stationary points on each curve.

a $y = x^2 + 2x$ **b** $y = 5x^2 - 4x + 1$ **c** $y = x^3 - 3x + 4$

d $y = 4x^3 + 3x^2 + 2$ **e** $y = 2x + 3 + \frac{8}{x}$ **f** $y = x^3 - 9x^2 - 21x + 11$

g $y = \frac{1}{x} - 4x^2$ **h** $y = 2x^{\frac{3}{2}} - 6x$ **i** $y = 9x^{\frac{2}{3}} - 2x + 5$

6 Find the coordinates of any stationary points on each curve. By evaluating $\frac{d^2y}{dx^2}$ at each stationary point, determine whether it is a maximum or minimum point.

a $y = 5 + 4x - x^2$ **b** $y = x^3 - 3x$ **c** $y = x^3 + 9x^2 - 8$

d $y = x^3 - 6x^2 - 36x + 15$ **e** $y = x^4 - 8x^2 - 2$ **f** $y = 9x + \frac{4}{x}$

g $y = x - 6x^{\frac{1}{2}}$ **h** $y = 3 - 8x + 7x^2 - 2x^3$ **i** $y = \frac{x^4 + 16}{2x^2}$

7 Find the coordinates of any stationary points on each curve and determine whether each stationary point is a maximum, minimum or point of inflexion.

a $y = x^2 - x^3$ **b** $y = x^3 + 3x^2 + 3x$ **c** $y = x^4 - 2$

d $y = 4 - 12x + 6x^2 - x^3$ **e** $y = x^2 + \frac{16}{x}$ **f** $y = x^4 + 4x^3 - 1$

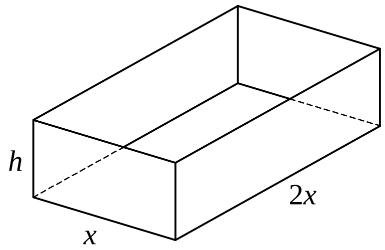
8 Sketch each of the following curves showing the coordinates of any turning points.

a $y = x^3 + 3x^2$ **b** $y = x + \frac{1}{x}$ **c** $y = x^3 - 3x^2 + 3x - 1$

d $y = 3x - 4x^{\frac{1}{2}}$ **e** $y = x^3 + 4x^2 - 3x - 5$ **f** $y = (x^2 - 2)(x^2 - 6)$

C2 DIFFERENTIATION Worksheet B

1



The diagram shows a baking tin in the shape of an open-topped cuboid made from thin metal sheet. The base of the tin measures x cm by $2x$ cm, the height of the tin is h cm and the volume of the tin is 4000 cm^3.

a Find an expression for h in terms of x.

b Show that the area of metal sheet used to make the tin, A cm^2, is given by

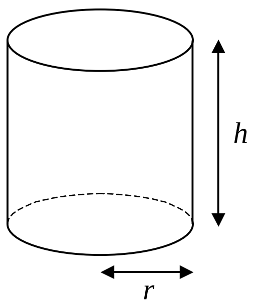
$$A = 2x^2 + \frac{12\,000}{x}.$$

c Use differentiation to find the value of x for which A is a minimum.

d Find the minimum value of A.

e Show that your value of A is a minimum.

2



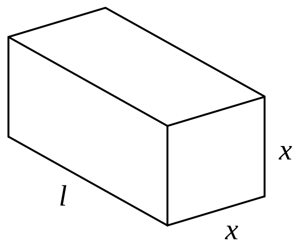
The diagram shows a closed plastic cylinder used for making compost. The radius of the base and the height of the cylinder are r cm and h cm respectively and the surface area of the cylinder is 30 000 cm^2.

a Show that the volume of the cylinder, V cm^3, is given by

$$V = 15\,000r - \pi r^3.$$

b Find the maximum volume of the cylinder and show that your value is a maximum.

3



The diagram shows a square prism of length l cm and cross-section x cm by x cm.
Given that the surface area of the prism is k cm^2, where k is a constant,

a show that $l = \frac{k - 2x^2}{4x}$,

b use calculus to prove that the maximum volume of the prism occurs when it is a cube.

C2 DIFFERENTIATION Worksheet C

1 $$f(x) \equiv 2x^3 + 5x^2 - 1.$$

a Find $f'(x)$.

b Find the set of values of x for which $f(x)$ is increasing.

2 The curve C has the equation $y = x^3 - x^2 + 2x - 4$.

a Find an equation of the tangent to C at the point $(1, -2)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Prove that the curve C has no stationary points.

3 A curve has the equation $y = \sqrt{x} + \frac{4}{x}$.

a Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve and determine its nature.

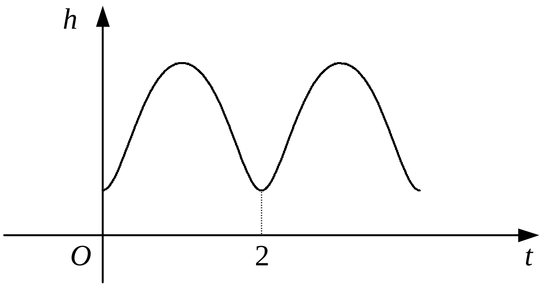
4 $$f(x) \equiv x^3 + 6x^2 + 9x.$$

a Find the coordinates of the points where the curve $y = f(x)$ meets the x-axis.

b Find the set of values of x for which $f(x)$ is decreasing.

c Sketch the curve $y = f(x)$, showing the coordinates of any stationary points.

5



The graph shows the height, h cm, of the letters on a website advert t seconds after the advert appears on the screen.

For t in the interval $0 \leq t \leq 2$, h is given by the equation

$$h = 2t^4 - 8t^3 + 8t^2 + 1.$$

For larger values of t, the variation of h over this interval is repeated every 2 seconds.

a Find $\frac{dh}{dt}$ for t in the interval $0 \leq t \leq 2$.

b Find the rate at which the height of the letters is increasing when $t = 0.25$

c Find the maximum height of the letters.

6 The curve C has the equation $y = x^3 + 3kx^2 - 9k^2x$, where k is a non-zero constant.

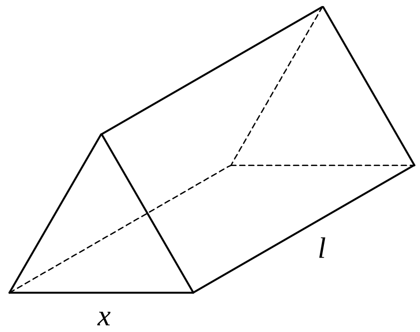
a Show that C is stationary when

$$x^2 + 2kx - 3k^2 = 0.$$

b Hence, show that C is stationary at the point with coordinates $(k, -5k^3)$.

c Find, in terms of k, the coordinates of the other stationary point on C.

7



The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is l cm.

Given that the volume of the prism is 250 cm^3,

a find an expression for l in terms of x,

b show that the surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x}).$$

Given that x can vary,

c find the value of x for which A is a minimum,

d find the minimum value of A in the form $k\sqrt{3}$,

e justify that the value you have found is a minimum.

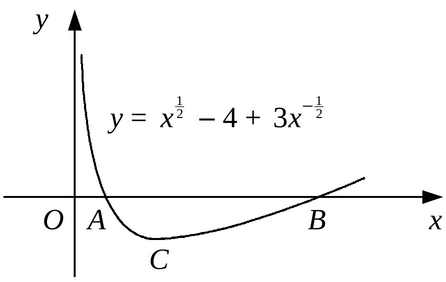
8 $$\mathrm{f}(x) \equiv x^3 + 4x^2 + kx + 1.$$

a Find the set of values of the constant k for which the curve $y = \mathrm{f}(x)$ has two stationary points.

Given that $k = -3$,

b find the coordinates of the stationary points of the curve $y = \mathrm{f}(x)$.

9



The diagram shows the curve with equation $y = x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}}$. The curve crosses the x-axis at the points A and B and has a minimum point at C.

a Find the coordinates of A and B.

b Find the coordinates of C, giving its y-coordinate in the form $a\sqrt{3} + b$, where a and b are integers.

10 $$\mathrm{f}(x) = x^3 - 3x^2 + 4.$$

a Show that $(x + 1)$ is a factor of $\mathrm{f}(x)$.

b Fully factorise $\mathrm{f}(x)$.

c Hence state, with a reason, the coordinates of one of the turning points of the curve $y = \mathrm{f}(x)$.

d Using differentiation, find the coordinates of the other turning point of the curve $y = \mathrm{f}(x)$.

C2 DIFFERENTIATION Worksheet D

1 $$f(x) \equiv 7 + 24x + 3x^2 - x^3.$$

a Find $f'(x)$. **(2)**

b Find the set of values of x for which $f(x)$ is increasing. **(4)**

2 The curve with equation $y = x^3 + ax^2 - 24x + b$, where a and b are constants, passes through the point P $(-2, 30)$.

a Show that $4a + b + 10 = 0$. **(2)**

Given also that P is a stationary point of the curve,

b find the values of a and b, **(4)**

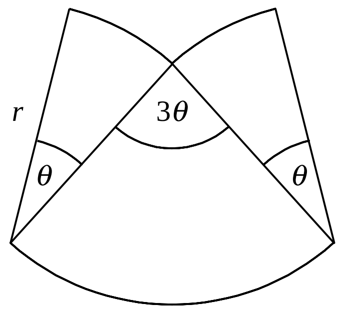
c find the coordinates of the other stationary point on the curve. **(3)**

3 $$f(x) \equiv x^2 + \frac{16}{x}, \quad x \neq 0.$$

a Find $f'(x)$. **(2)**

b Find the coordinates of the stationary point of the curve $y = f(x)$ and determine its nature. **(6)**

4



The diagram shows a design to be used on a new brand of cat-food. The design consists of three circular sectors, each of radius r cm. The angle of two of the sectors is θ radians and the angle of the third sector is 3θ radians as shown.

Given that the area of the design is 25 cm^2,

a show that $\theta = \frac{10}{r^2}$, **(3)**

b find the perimeter of the design, P cm, in terms of r. **(3)**

Given that r can vary,

c find the value of r for which P takes it minimum value, **(4)**

d find the minimum value of P, **(1)**

e justify that the value you have found is a minimum. **(2)**

5 The curve C has the equation

$$y = 2x - x^{\frac{3}{2}}, \quad x \geq 0.$$

a Find the coordinates of any points where C meets the x-axis. **(3)**

b Find the x-coordinate of the stationary point on C and determine whether it is a maximum or a minimum point. **(6)**

c Sketch the curve C. **(2)**

6 The curve $y = x^3 - 3x + 1$ is stationary at the points P and Q.

a Find the coordinates of the points P and Q. **(5)**

b Find the length of PQ in the form $k\sqrt{5}$. **(3)**

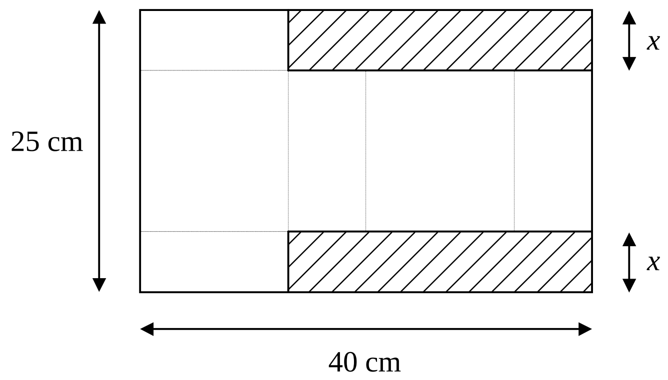
7 $$f(x) \equiv 2x - 5 + \frac{2}{x}, \quad x \neq 0.$$

a Solve the equation $f(x) = 0$. **(4)**

b Solve the equation $f'(x) = 0$. **(4)**

c Sketch the curve $y = f(x)$, showing the coordinates of any turning points and of any points where the curve crosses the coordinate axes. **(3)**

8



Two identical rectangles of width x cm are removed from a rectangular piece of card measuring 25 cm by 40 cm as shown in the diagram above. The remaining card is the net of a cuboid of height x cm.

a Find expressions in terms of x for the length and width of the base of the cuboid formed from the net. **(3)**

b Show that the volume of the cuboid is $(2x^3 - 65x^2 + 500x)$ cm^3. **(2)**

c Find the value of x for which the volume of the cuboid is a maximum. **(5)**

d Find the maximum volume of the cuboid and show that it is a maximum. **(3)**

9 **a** Find the coordinates of the stationary points on the curve

$$y = 2 + 9x + 3x^2 - x^3.$$ **(6)**

b Determine whether each stationary point is a maximum or minimum point. **(2)**

c State the set of values of k for which the equation

$$2 + 9x + 3x^2 - x^3 = k$$

has three solutions. **(2)**

10 $$f(x) = 4x^3 + ax^2 - 12x + b.$$

Given that a and b are constants and that when $f(x)$ is divided by $(x + 1)$ there is a remainder of 15,

a find the value of $(a + b)$. **(2)**

Given also that when $f(x)$ is divided by $(x - 2)$ there is a remainder of 42,

b find the values of a and b, **(3)**

c find the coordinates of the stationary points of the curve $y = f(x)$. **(6)**